I. Introduction:

This document is a revised version of the scope and sequence guide created last year and the year before by elementary mathematics workgroup teams. These revisions reflect feedback from teacher leaders and teachers collected over this past year.

This Grade 2 scope and sequence guide reflects the following Common Core Shifts for Mathematics:

1. **Focus** strongly where the Standards focus
2. **Coherence**: Think across grades and link to major topics within grades
3. **Rigor**: Require fluency, application, and deep understanding

It also reflects *full alignment* with the 2011 MCF for Mathematics, which contains the CCSS for Mathematics, and the PARCC Model Content Framework for Mathematics.

In Grade 2, instructional time focuses on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. (See 2011 MCF for Mathematics, page 34.) By the end of the year, students should be proficient with the Grade 2 Content and Practice Standards.

II. Grade 2 Standards for Mathematical Practice:

*The Common Core State Standards for Mathematical Practice* are practices expected to be integrated into every mathematics lesson for all students grades K-12. Below are draft elaborations of the mathematical practice standards for the elementary grades. The original document can be found on the Illustrative Mathematics webpage at [http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards/](http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards/).

**MP 1 - Make sense of problems and persevere in solving them.**

Mathematically proficient students at the elementary grades explain to themselves the meaning of a problem, look for entry points to begin work on the problem, and plan and choose a solution pathway. For example, young students might use concrete objects or pictures to show the actions of a problem, such as counting out and joining two sets to solve an addition problem. If students are not at first making sense of a problem or seeing a way to begin, they ask questions that will help them get started. As they work, they continually ask themselves, “Does this make sense?” When they find that their solution pathway does not make sense, they look for another pathway that does. They may consider simpler forms of the original problem; for example, to solve a problem involving multidigit numbers, they might first consider similar problems that involve multiples of ten or one hundred. Once they have a solution, they look back at the problem to determine if the solution is reasonable and accurate. They often check their answers to problems using a different method or approach. Mathematically proficient students consider different representations of the problem and different solution pathways, both their own and those of other students, in order to identify and analyze correspondences among approaches. They can explain correspondences among physical models, pictures or diagrams, equations, verbal descriptions, tables, and graphs.
Mathematically proficient students can contextualize an abstract problem by placing it in a context they then use to make sense of the mathematical ideas. For example, when a student sees the expression 40 - 26, she might visualize this problem by thinking, if I have 26 marbles and Marie has 40, how many more do I need to have as many as Marie? Then, in that context, she thinks, 4 more will get me to a total of 30, and then 10 more will get me to 40, so the answer is 14. In this example, the student uses a context to think through a strategy for solving the problem, using the relationship between addition and subtraction and decomposing and recomposing the quantities. She then uses what she did in the context to identify the solution of the original abstract problem.

Mathematically proficient students can also make sense of a contextual problem and express the actions or events that are described in the problem using numbers and symbols. If they work with the symbols to solve the problem, they can then interpret their solution in terms of the context. For example, to find the area of the floor of a rectangular room that measures 10 m. by 12 m., a student might represent the problem as an equation, solve it mentally, and record the problem and solution as 10 x 12 = 120. He has decontextualized the problem. When he states at the end that the area of the room is 120 square meters, he has contextualized the answer in order to solve the original problem. Problems like this that begin with a context and are then represented with mathematical objects or symbols are also examples of modeling with mathematics (MP.4).

Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and reconsider their own arguments in response to the critiques of others.

Mathematically proficient students present their arguments in the form of representations, actions on those representations, and explanations in words (oral or written). In the elementary grades, arguments are often a combination of all three. Some of their arguments apply to individual problems, but others are about conjectures based on regularities they have noticed across multiple problems (see MP.8, Look for and express regularity in repeated reasoning). As they articulate and justify generalizations, students consider to which mathematical objects (numbers or shapes, for example) their generalizations apply. For example, young students may believe a generalization about the behavior of addition applies to positive whole numbers less than 100 because those are the numbers with which they are currently familiar. As they expand their understanding of the number system, they may reexamine their conjecture for numbers in the hundreds and thousands. In upper elementary grades, students return to their conjectures and arguments about whole numbers to determine whether they apply to fractions and decimals. For example, students might make an argument based on an area representation of multiplication to show that the distributive property applies to problems involving fractions.

Mathematically proficient students can listen to or read the arguments of others, decide whether they make sense, ask useful questions to clarify or improve the arguments, and build on those arguments. They can communicate their arguments, compare them to others, and reconsider their own arguments in response to the critiques of others.
MP 4 - Model with mathematics.

When given a problem in a contextual situation, mathematically proficient students at the elementary grades can identify the mathematical elements of a situation and create a mathematical model that shows those mathematical elements and relationships among them. The mathematical model might be represented in one or more of the following ways: numbers and symbols, geometric figures, pictures or physical objects used to abstract the mathematical elements of the situation, or a mathematical diagram such as a number line, a table, or a graph, or students might use more than one of these to help them interpret the situation. For example, when students are first studying an operation such as addition, they might arrange counters to solve problems such as this one: there are seven animals in the yard, some are dogs and some are cats, how many of each could there be? They are using the counters to model the mathematical elements of the contextual problem—that they can split a set of 7 into a set of 3 and a set of 4. When they learn how to write their actions with the counters in an equation, \(4 + 3 = 7\), they are modeling the situation with numbers and symbols. Similarly, when students encounter situations such as sharing a pan of cornbread among 6 people, they might first show how to divide the cornbread into 6 equal pieces using a picture of a rectangle. The rectangle divided into 6 equal pieces is a model of the essential mathematical elements of the situation. When the students learn to write the name of each piece in relation to the whole pan as \(1/6\), they are now modeling the situation with mathematical notation.

Mathematically proficient students are able to identify important quantities in a contextual situation and use mathematical models to show the relationships of those quantities, particularly in multistep problems or problems involving more than one variable. For example, if there is a Penny Jar that starts with 3 pennies in the jar, and 4 pennies are added each day, students might use a table to model the relationship between number of days and number of pennies in the jar. They can then use the model to determine how many pennies are in the jar after 10 days, which in turn helps them model the situation with the expression, \(4 \times 10 + 3\).

Mathematically proficient students use their model to analyze the relationships and draw conclusions. They interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. As students model situations with mathematics, they are choosing tools appropriately (MP.5). As they decontextualize the situation and represent it mathematically, they are also reasoning abstractly (MP.2).

MP 5 - Use appropriate tools strategically.

Mathematically proficient students at the elementary grades consider the tools that are available when solving a mathematical problem, whether in a real-world or mathematical context. These tools might include physical objects (cubes, geometric shapes, place value manipulatives, fraction bars, etc.), drawings or diagrams (number lines, tally marks, tape diagrams, arrays, tables, graphs, etc.), paper and pencil, rulers and other measuring tools, scissors, tracing paper, grid paper, virtual manipulatives or other available technologies. Proficient students are sufficiently familiar with tools appropriate for their grade and areas of content to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained from their use as well as their limitations.

Mathematically proficient students choose tools that are relevant and useful to the problem at hand. These include such tools as are mentioned above as well as mathematical tools such as estimation or a particular strategy or algorithm. For example, in order to solve \(3/5 - 1/2\), a student might recognize that knowledge of equivalents of \(1/2\) is an appropriate tool: since \(1/2\) is equivalent to \(2\ 1/2\) fifths, the result is \(1/2\) of a fifth or \(1/10\).

This practice is also related to looking for structure (MP.7), which often results in building mathematical tools that can be then used to solve problems.

MP 6 - Attend to precision.

Mathematically proficient students at the elementary grades communicate precisely to others. They start by using everyday language to express their mathematical ideas, realizing that they need to select words with clarity and specificity rather than saying, for example, “it works” without explaining what “it” means. As they encounter the ambiguity of everyday terms, they come to appreciate, understand, and use mathematical vocabulary. Once young students become familiar with a mathematical idea or object, they are ready to learn more precise mathematical terms to describe it. In using representations, such as pictures, tables, graphs, or diagrams, they use appropriate labels to communicate the meaning of their representation.

When making mathematical arguments about a solution, strategy, or conjecture (see MP.3), mathematically proficient students learn to craft careful explanations that communicate their reasoning by referring specifically to each important mathematical element, describing the relationships among them, and connecting their words clearly to their representations.
Elementary students learn to use mathematical symbols correctly and can describe the meaning of the symbols they use. In particular, they understand that the equal sign denotes that two quantities have the same value, and can use it flexibly to express equivalences. For example, the equivalence of $8$ and $5 + 3$ can be written both as $5 + 3 = 8$ and $8 = 5 + 3$. Similarly, the equivalence of $6 + 2$ and $5 + 3$ is expressed as $6 + 2 = 5 + 3$.

When measuring, mathematically proficient students use tools and strategies to minimize the introduction of error. From Kindergarten on, they count accurately, using strategies so that they include each object once and only once without losing track. Mathematically proficient students calculate accurately and efficiently and use clear and concise notation to record their work.

**MP 7 - Look for and make use of structure.**

Mathematically proficient students at the elementary grades use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (MP.8). For example, when younger students recognize that adding 1 results in the next counting number, they are identifying the basic structure of whole numbers. When older students calculate $16 \times 9$, they might apply the structure of place value and the distributive property to find the product: $16 \times 9 = (10 + 6) \times 9 = (10 \times 9) + (6 \times 9)$. To determine the volume of a $3 \times 4 \times 5$ rectangular prism, students might see the structure of the prism as five layers of $3 \times 4$ arrays of cubes.

**MP 8 - Look for and express regularity in repeated reasoning.**

Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities. For example, students might notice a pattern in the change to the product when a factor is increased by 1: $5 \times 7 = 35$ and $5 \times 8 = 40$ — the product changes by $5; 9 \times 4 = 36$ and $10 \times 4 = 40$ — the product changes by 4. Students might then express this regularity by saying something like, “When you change one factor by 1, the product increases by the other factor.” Younger students might notice that when tossing two-color counters to find combinations of a given number, they always get what they call “opposites”—when tossing 6 counters, they get 2 red, 4 yellow and 4 red, 2 yellow and when tossing 4 counters, they get 1 red, 3 yellow and 3 red, 1 yellow. Mathematically proficient students formulate conjectures about what they notice, for example, that when 1 is added to a factor, the product increases by the other factor; or that, whenever they toss counters, for each combination that comes up, its “opposite” can also come up. As students practice articulating their observations, they learn to communicate with greater precision (MP.6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (MP.3).

**III. Student Learning Tasks (SLTs):**

One of the priorities of the BPS Mathematics Office is to support the use of formative assessment as a powerful instructional tool. In this spirit, the Mathematics Office is creating three SLTs for Grade 2 that mirror PARCC Performance-Based Assessments (PBAs). The purpose of these tasks is to provide teachers and students with opportunities to engage with rich problems that are aligned with the major work of the grade. These tasks are to be administered in late October, early February, and the middle of April. In addition, within each Units of Study in this scope and sequence guide you will find rich tasks that are aligned with the standards addressed in each unit that are to be used formatively by teachers to assess student understanding and provide instruction that moves learning forward. Please refer to the *NCTM Research Brief Five “Key Strategies” for Effective Formative Assessment* to learn more about how these tasks can be used as part of a formative assessment process: [http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Research_brief_04_-_Five_Key%20Strategies.pdf](http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Research_brief_04_-_Five_Key%20Strategies.pdf)
IV. Curriculum Materials and Supporting Resources including Number Talks

The following curriculum materials are specifically referenced in this Grade 2 scope and sequence guide:

UNIT OF STUDY 1: ESTABLISHING GRADE 2 Routines AND NUMBER FLuENCY
Counting, Coins and Combinations, Grade 2 Investigations, U1
Investigations and the Common Core State Standards Guide
Differentiation and Intervention Guide
Time and Money Appendices (see Mathematics Department on the Aspen/SIS web page at https://sis.mybps.org/aspen/index.html)

UNIT OF STUDY 2: UNDERSTANDING PLACE VALUE 1
Stickers, Strings, and Story Problems Grade 2 Investigations U3
How Many Tens? How Many Ones? Grade 2 Investigations U6
Appendix (see Mathematics Department on the Aspen/SIS web page at https://sis.mybps.org/aspen/index.html)

UNIT OF STUDY 3: SOLVING STORY PROBLEMS
Stickers, Number Strings and Story Problems, Grade 2 Investigations U3
Differentiation and Intervention Guide
Investigations and the Common Core State Standards
Appendix (see Mathematics Department on the Aspen/SIS web page https://sis.mybps.org/aspen/index.html)

UNIT OF STUDY 4: ODD AND EVEN NUMBERS AND STORY PROBLEMS
Stickers, Number Strings and Story Problems, Grade 2 Investigations U3
Partners, Teams and Paper Clip, Grade 2 Investigations U8
Investigations and the Common Core State Standards Guide
Appendix (see Mathematics Department on the Aspen/SIS web page https://sis.mybps.org/aspen/index.html)

UNIT OF STUDY 5: UNDERSTANDING PLACE VALUE 2
Investigations and the Common Core State Standards Guide U6 and U8
Trading Stickers and Combining Coins, Grade 3 Investigations U1
Appendix (see Mathematics Department on the Aspen/SIS web page https://sis.mybps.org/aspen/index.html)

UNIT OF STUDY 6: GEOMETRY AND EQUAL SHARES
Shapes, Blocks, and Symmetry, Grade 2 Investigations U2
Parts of a Whole, Parts of a Group, Grade 2 Investigations U7
Appendix (see Mathematics Department on the Aspen/SIS web page https://sis.mybps.org/aspen/index.html)

UNIT OF STUDY 7: DATA, MEASUREMENT AND TIME
Measuring Length and Time, Grade 2 Investigations U9
Pockets, Teeth and Favorite Things, Grade 2 Investigations U4
Investigations and the Common Core State Standards
Appendix (see Mathematics Department on the Aspen/SIS web page https://sis.mybps.org/aspen/index.html)
STRENGTHENING FLUENCY WITH NUMBER TALKS

The 2011 MCF for Mathematics names standards for fluency with single-digit combinations in addition, subtraction, multiplication, and division at different grade levels.

The word fluent is used in the Standards to mean “fast and accurate.” Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., “adding 0 yields the same number”), and knowing some answers from the use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency will be a mixture of these kinds of thinking, which may differ across students. . . The extensive work relating addition and subtraction means that subtraction can frequently be solved by thinking of the related addition, especially for smaller numbers. It is also important that these patterns, strategies and decompositions still be available in Grade 3 for use in multiplying and dividing and in distinguishing adding and subtracting from multiplying and dividing. The important press toward fluency should also allow students to fall back on earlier strategies when needed. By the end of the K–2 grade span, students have sufficient experience with addition and subtraction to know single-digit sums from memory; as should be clear from the foregoing, this is not a matter of instilling facts divorced from their meanings, but rather as an outcome of a multi-year process that heavily involves the interplay of practice and reasoning (excerpt from K, Counting and Cardinality; K–5, Operations and Algebraic Thinking CCSS Progression).

The framework defines fluency with multi-digit numbers, including decimals, and fraction as “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (2011 MCF for Mathematics, p. 15). The framework also specifies the following Grade 2 fluency standards:

2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two one-digit numbers.
2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Number Talks are designed to support proficiency with grade level fluency standards. The goal of Number Talks is for students to compute accurately, efficiently, and flexibly. In addition to developing efficient computation strategies, Number Talks encourages students to make sense of mathematics, communicate mathematically, and reason and prove solutions. (See Number Talks by Sherry Parrish, Math Solutions 2010.) It is expected that Number Talks take place for at least ten minutes at least three times a week.

The key components of successful Number Talks include the following:

1. A safe and accepting classroom environment and mathematical community
2. Classroom discussions (PROTOCOL)
   - Teacher provides the problem.

Additional supporting resources include the following:
- Illustrative Mathematics Project: https://www.illustrativematics.org
- Inside Mathematics: http://www.insidemathematics.org
- Engage NY curricular resources: http://www.engageny.org/common-core-curriculum-assessments
- Achieve the Core resources: http://achievethecore.org

Boston Public Schools Elementary Mathematics Department
Grade 2 Scope and Sequence, 2015-2016. Last updated 8.12.15
• Teacher provides students opportunity to solve problem mentally.
• Students show a visual cue when they are ready with a solution. Students signal if they have solved it in more than one way too. (Quiet form of acknowledgement allows time for students to think, while the process continues to challenge those that are already have an answer.)
• Teacher calls for answers. S/he collects all answers- correct and incorrect- and records answers.
• Students share strategies and justifications with peers.

3. The teacher’s role as a “facilitator, questioner, listener, and learner”
4. Use of mental math to increase efficiency and knowledge of number relationships
5. Purposeful computation problems that support mathematical goals in number and operations

The BPS Mathematics Department has provided resources to schools to support the Number Talks routine. However, these are only meant to be resources. The purpose of Number Talks is for each teacher to use the protocol to address the needs of his or her students. Crafting problems that guide students to focus on mathematical relationships is an essential part of number talks that is used to build mathematical understanding and knowledge. The teacher’s goals and purposes for the number talk should determine the numbers and operations that are chosen. Careful planning before the number talk is necessary to design ‘just right’ problems for students. (See Number Talks, p. 14.) Teachers are encouraged to design their own Number Talks based upon informal and formal assessment data. For example, at the beginning of the 3rd grade, teachers might want to initially revisit two digit plus two-digit addition and subtraction from previous grades. Using small numbers serves two purposes: 1) students can focus on the nuances of the strategy instead of the magnitude of the numbers, 2) students are able to build confidence in their mathematical abilities. (See Number Talks, p. 183.)

**Areas to consider when selecting Number Talk Problems** (see Number Talks, p. 373):

- **Over-generalizations.** When students are investigating which strategies work with different operations, they often over-generalize and try to apply their generalizations to all operations. An example is when students are convinced that compensation works with addition and then assume it will also work with subtraction, multiplication, or division.
- **Inefficient strategies.** Sometimes students become more focused on a specific strategy and ignore efficiency. If you have given them a problem that lends itself to using landmark numbers or compensation, such as 199 + 199, yet the majority of your students solve this either with the standard U.S. algorithm or by breaking it apart by place value, you would want to craft problems to address this issue.
- **Evidence from exit cards.** Exit cards are an excellent way to keep a pulse on students’ understanding and use of strategies. If students struggle with a specific type of problem or operation on their exit cards, this would guide the types of problems and strategies for the next day’s number talk.

**Examples available as video links:**

- **Kindergarten:** *Quick Images: Visualizing Number Combinations:* [https://www.teachingchannel.org/videos/visualizing-number-combinations](https://www.teachingchannel.org/videos/visualizing-number-combinations)
- **Grade 3:** *Reasoning About Multiplication and Division:* [https://www.teachingchannel.org/videos/multiplication-division-in-the-core](https://www.teachingchannel.org/videos/multiplication-division-in-the-core)
- **Grade 3:** *Grade 3 Number Talk:* [http://www.mathsolutions.com/videopage/videos/Final/Classroom_NumberTalk_Gr3.swf](http://www.mathsolutions.com/videopage/videos/Final/Classroom_NumberTalk_Gr3.swf)
- **Grade 4:** *Reasoning About Division:* [https://www.teachingchannel.org/videos/common-core-teaching-division](https://www.teachingchannel.org/videos/common-core-teaching-division)
V. Family and Student Engagement

Consider your role in engaging and partnering with families to support a student’s academic success. The following BPS Family and Student Engagement Standards (see http://bpsfamilies.org/about-ofse/family-student-engagement-standards) serve as a guide to support the establishment of effective family and student engagement practice in every BPS school.

1. Welcoming All Families and Students into the School Community: Families and students are active participants in the life of the school, and feel welcomed, valued and connected to each other, to school staff, and to what students are learning and doing in class. Suggestion: Plan a parent-child math morning or night in the fall to let parents become familiar with the important math ideas of your grade level.

2. Communicating Effectively: Families, students and school staff engage in regular two-way meaningful communication about student learning and progress. Suggestion: Survey parents at the beginning of year about the best way to communicate (e.g., email, phone, text).

3. Supporting Student Success: Families and school staff continuously collaborate to support student learning and healthy development both at home and school, and have regular opportunities to strengthen their knowledge and skills to do so effectively. School staff ensures that students understand expectations, academic requirements and have access to regular progress reports. Suggestion: Provide information about a student’s successes as well as concerns. Have student share a key task or problem that shows perseverance and learning.

4. Speaking up for Every Student: Families are empowered to be advocates for their own and other children to ensure that all students are treated fairly and both students and families understand their rights. Suggestion: Include a ‘listening’ conference in the fall to hear parent concerns and hopes for the year.

Resources that support family and student engagement include the following:

Investigations is the primary elementary math curriculum used in BPS. The Student Math Handbooks offer a valuable reference to the math words and ideas introduced in the curriculum units, as well as game directions for the games in that grade. Students can take this resource home for reference while doing homework and playing math games with their families and/or as a reference for families to better understand the work their children are doing in class. References to the Student Math Handbooks are included in every Session Follow-Up section and on every homework sheet and practice page. This resource is also available in Spanish and online, through Pearson's SuccessNet.

Parent Roadmaps to the Common Core Standards - Mathematics: http://www.cgcs.org/Page/244
The Council of the Great City Schools' parent roadmaps in mathematics provide guidance to parents about what their children will be learning and how they can support that learning in grades K-8. The parent roadmaps for each grade level also provide three-year snapshots showing how selected standards progress from year to year so that students will be college and career ready upon their graduation from high school. They are available in English and Spanish.

Learn Zillion: http://learnzillion.com/
Learn Zillion offers a free and growing set of Math resources for grades 2-12 that have been developed by expert teachers directly from the Common Core State Standards. Parents can create a free account and access lessons and tutorials for themselves and/or their children.
VI. Abbreviations Used in this Scope and Sequence Guide:

**2011 MCF for Mathematics** is the 2011 Massachusetts Curriculum Framework for Mathematics.

**CCSS** is the Common Core State Standards.

The **CCSS Guide** is the *Investigations and the Common Core State Standards* booklet that helps align our curriculum resources to the new MCF standards. The lessons for these sessions end in a letter, A or B.

The **Progressions** documents are documents the authors of the CCSS for mathematics created to help teachers and school districts understand the depth and breadth of the CCSS mathematics content standards and how they develop over time.

**SAB** is the *Investigations* Student Activity Book.
## UNIT OF STUDY 1: ESTABLISHING GRADE 2 ROUTINES AND NUMBER FLUENCY

**September 8 – September 23**

**Primary Curricular Resource:**
*Grade 2, Unit 1, Coins and Combinations*
*Investigations and the Common Core State Standards Guide*
*Differentiation and Intervention Guide*

Time and Money Appendices (see Mathematics Department on the Aspen/SIS web page at [https://sis.mybps.org/aspen/index.html](https://sis.mybps.org/aspen/index.html))

**Estimated Instructional Time:** 14 days

### Overarching Questions:
- How do I establish routines and foster a productive, respectful and reflective mathematical community?
- How can I support the development of number fluency through the explicit use of mental strategies and representations?
- How do students make sense of and solve story problems by visualizing, retelling and modeling?

### Instructional Notes:

#### Building a Mathematical Community
Use this time to begin to build a math community where discourse can lead to deeper understanding, help students feel safe to make mistakes, and facilitate development of mathematical ideas. Based on *Number Sense Routines*, by Jessica Shumway, here are four student skills to focus on as you begin the year:

1. How to Explain Your Thinking
2. How to Be an Active Listener
3. How to Have a Conversation
4. How to be Supportive of Your Fellow Mathematicians

*When teaching word problems, engage students in sense-making and avoid using key words for the following reasons:*

1. **Key words can be misleading.** Some key words typically mean addition or subtraction. But not always. Consider “There were 4 jackets left on the playground on Monday and 5 jackets left on the playground on Tuesday. How many jackets were left on the playground?” The word "left" in this problem does not mean subtract.

2. **Many problems have no key words.** For example, “How many legs do 7 elephants have?” does not have a key word. However, any 1st or 2nd grader should be able to solve the problem by thinking and drawing a picture or building a model.

3. **It sends a bad message.** The most important strategy when solving a problem is to make sense of the problem and to think. Key words encourage students to ignore meaning and look for a formula. Mathematics is about meaning (Van de Walle, 2012).

*From Howard County Public Schools, [https://grade1commoncoremath.wikispaces.hcpss.org/Problem+Solving](https://grade1commoncoremath.wikispaces.hcpss.org/Problem+Solving)*

**Differentiation Guide:** The Differentiation Guide can be useful throughout the year. For this unit, you may want to use pages 2-17.
**Methods used for solving single-digit addition and subtraction problems:**

**Students should enter second grade proficient with counting on and having had experiences with Level 3 Strategies.**

Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful using addition to solve for a subtraction situation.

- **Level 1:** Direct Modeling by Counting All or Taking Away
  - **Addition:** Count On
  - **Subtraction:** To solve a subtraction problem, change to missing addend problem and count on to reach subtrahend.

- **Level 2:** Counting On
  - **Addition:** Count On
  - **Subtraction:** To solve a subtraction problem, change to missing addend problem and count on to reach subtrahend.

- **Level 3:** Recompose (Convert to an Easier Equivalent Problem)
  - **Make a ten (general):** one addend breaks apart to make 10 with the other addend
  - **Make a ten (from 5's within each addend):**
  - **Doubles ± n:**
    
  
  (from Counting and Cardinality; K-5, Operations and Algebraic Thinking Progressions Document at commoncoretools.wordpress.com
  <http://commoncoretools.wordpress.com>, see pp. 36-39 for descriptions and examples of Levels and Strategies)

**Time and Money Activities:** Incorporate time and money activities into your morning meeting, math workshop, or another part of your day approximately twice a week. See the Time and Money Appendices for games and activities throughout the year.

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<tr>
<th>Concepts developed in this unit:</th>
<th>Prior knowledge expected:</th>
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| This is an opportunity to foster a classroom community through establishing classroom routines, supporting the development of number fluency, having mathematical conversations, and having students make sense of the action of and solve story problems. Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding. Students need to understand mathematics and use it effectively. To achieve mathematical understanding students should have a balance of mathematical procedures and conceptual understanding. Students should be engaged in doing meaningful mathematics, discussing mathematical ideas, and applying mathematics in interesting, thought-provoking situations. | **Operations and Algebraic Thinking**

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use mental strategies such as counting on; making ten (e.g., 8+6=8+2+4=10+4=14); decomposing a number leading to ten (e.g., 13-4=13-3-1=10-1=9); using the relationship between addition and subtraction (e.g., knowing that 8+4=12, one knows 12-8=4); and creating equivalent but easier or known sums (e.g., 6+7 by creating the known equivalent 6+6+1=12+1=13)

**Measurement and Data**

1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks.

MA.5. Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel) Find equivalent values (e.g., a nickel is equivalent to 5 pennies). Use appropriate notation (e.g., 69 ¢). Use values of coins in the solutions of problems.
Student Learning Outcomes:

Operations and Algebraic Thinking
Use addition and subtraction to solve word problems involving situations of adding to and taking from, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem. (2.OA.1)
Add and subtract within 20 using mental strategies. (2.OA.2)
Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (2.OA.4)

Number and Operations in Base Ten
Use place value understanding and properties of operations to add and subtract.
Explain why addition and subtraction strategies work, using place value and the properties of operations. (2.NBT.9)

Measurement and Data
Tell and write time from analog and digital clocks. (2.MD.6)
Know the relationships of time, including seconds in a minute, minutes in an hour, hours in a day, days in a week, a month, and a year; and weeks in a month and a year. (MA.7.a)
Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and cent symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? (2.MD.8)

Geometry
Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (2.G.2)

<table>
<thead>
<tr>
<th>MA 2011</th>
<th>After completing each investigation, students will be able to:</th>
<th>Days</th>
<th>Primary Curriculum Resource</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.OA.4 2.OA.2 2.OA.1 2.OA.2 MA.2.a 2.MD.7 2.MD.8</td>
<td>• Establish use of tools, routines, and expectations for math class. • Use and understand the structure of the number line and 100 chart Use academic language such as: analog clock, digital clock, o’clock, greater than, less than.</td>
<td>2</td>
<td>Begin to use BPS Number Talks Protocol Packet (on Elementary math website) for 10 minute routine 3 time each week For this unit of study, start each math block with Number Talks for 10-15 minutes to establish classroom routine and community. Choose a Number Talk that all students will be able to access. Gr. 2, Unit 1: Counting, Coins, and Combinations 1.3 The Number Line 1.4 The 100 Chart Math Workshop –Add one center for telling time to the half hour (review from first grade): Time Concentration, see Time Appendix</td>
</tr>
<tr>
<td>1</td>
<td>The Beginning of Year assessment can take place during the first full week. (If the pre-assessment does not take a full day, you may use Number Talks and/or some Math Workshop activities that are part of this unit.)</td>
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<tr>
<td>2.NBT.9 &amp; 2.G.2</td>
<td>Start each math block with 10-15 minutes of Number Talks.</td>
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<td></td>
<td>• Count sets of up to 60 objects.</td>
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<td></td>
<td>• Identify coins and their values.</td>
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<tr>
<td></td>
<td>• Identify and use coin values</td>
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<td></td>
<td>Use academic language such as: cents, penny, nickel, dime, quarter.</td>
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<td>2</td>
<td>Introduce Money Routines</td>
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<td></td>
<td><strong>Day 1: Collect 50 Cents</strong></td>
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<td><strong>Day 2: Spend 50 Cents</strong></td>
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<td></td>
<td>Math Workshop - Include telling time to the hour and half hour.</td>
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<td><strong>See Time Appendix</strong></td>
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<td></td>
<td>Extension: Provide word problems with money.</td>
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<td><strong>See Money Appendix</strong></td>
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<tr>
<td>Develop fluency with the subtraction facts related to the Plus 1, Plus 2 and Make 10 addition combinations</td>
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<tr>
<td>1</td>
<td><strong>Investigations and the Common Core State Standards</strong> Book Unit 2 - 1.1A Subtraction Facts</td>
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<tr>
<td></td>
<td>• Visualize, retell and model the action of addition and subtraction (removal) situations.</td>
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<td></td>
<td>• Combine two quantities with totals up to 45. Subtract a quantity from a whole of up to 30.</td>
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<td></td>
<td>• Record strategies for solving addition and subtraction story problems. Connect standard notation for addition and subtraction to the quantities and actions that the signs and symbols represent.</td>
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<td></td>
<td>• Use a rectangular array to model doubling.</td>
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<td></td>
<td>Use academic language such as: parts, whole, equation, plus sign, equal sign, more, same, fewer, minus sign.</td>
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</tbody>
</table>

There is no End-of-Unit Assessment for this Unit of Study.
UNIT OF STUDY 2: UNDERSTANDING PLACE VALUE 1

Primary Curricular resource
Grade 2, Unit 3-Stickers, Strings, and Story Problems
Grade 2, Unit 6-How Many Tens? How Many Ones?
Unit of Study 2 Appendix (see Mathematics Department on the Aspen/SIS web page at https://sis.mybps.org/aspen/index.html)

Estimated Instructional Time: 29 Days
September 24-November 4

Overarching Questions:
• How do we read and write numerals to 100 using base-ten numerals, number names and expanded form?
• How can two digit numbers be decomposed in tens and ones in different ways? (e.g., 37 can be 3 tens, 7 ones or 2 tens, 17 ones or 1 ten 27 ones or 37 ones)
• How do we add and subtract numbers within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction?
• How can we use the number line as a tool to show a strategy?

Instructional Notes:

Place Value:
This unit helps students to understand that '100' is the same amount as 10 tens, building on their thinking of groups of numbers as units rather as about numbers one by one. This lays the essential foundation for work with the structure of the base-ten system in future grades.

• Students use the sticker notation and base ten models as manipulatives to help solve problems and build their understanding of our base ten system, becoming fluent in composing by hundreds, tens, and ones. For some students, some work with bundling group-able objects such as popsicle sticks or cubes. may be necessary until they have a solid grasp of place value.


• Encourage students to actively and accurately represent their thinking. This is a useful assessment tool and can also help consolidate learning. For example, when a student uses the number line as a tool, notice what magnitudes of jumps were used and ask questions about how they chose those jumps.
• The U.S. standard algorithm of carrying or borrowing is not an expectation in Second Grade and may interfere with students' practice with reasoning about number and operations. Encourage students to use strategies that make sense to them and be able to explain why it works. Students are not expected to add and subtract whole numbers using standard algorithms until the end of Fourth Grade.

Count Around the Circle and Choral Counting
Use Count Around the Circle during the launch to count by tens to 200 or more and by hundreds to 1000. This is routine that to highlight a counting sequence – for example, count by tens starting at thirty-two. Each person says a number as they go around the circle. (For example, the first person says, “Thirty-two,” the second person says, “Forty-two,” the next person says, “Fifty-two,” and so on. You can have students count both forward and back to practice mentally adding and subtracting by 10 or 100.

In the choral counting routine, the class counts aloud a number sequence all together.

For an example of choral counting, go to https://www.teachingchannel.org/videos/teaching-number-patterns

**Practicing Addition Combinations and Related Subtraction Facts:**
Throughout this unit students should continue to review and practice their sets of addition combinations and the related subtraction facts. This practice can be assigned as additional homework, a Math Workshop activity, or something students do when they have a few minutes after completing another activity. Pages C50 and C57 allow you to individualize this practice according to student needs at any time during this unit.

**Concepts developed in this unit:**
Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens and ones as well as number relationships involving these units, including comparing.

Students understand multi-digit numbers (up to 100) written in base-ten notation, recognizing that the digits in each place represent amounts of hundreds, tens or ones. Students use their understanding of addition to develop fluency with addition and subtraction within 100.

They solve problems within 100 by applying their understanding of models for addition and subtraction, and they develop, discuss and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations.

**Prior knowledge expected:**

**Number in Operations in Base Ten**

1.NBT.2. Understand that the two digits of a 2-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. 10 can be thought of as a bundle of ten ones – called a “ten”.
   b. The numbers from 11 to 19 are composed of a ten and a one, two, three, four, five, six, seven, eight, or nine one.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refers to 1, 2, 3, 4, 5, 6, 7, 8, or 9 tens (and 0 ones).

1.NBT.4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones, and sometimes it is necessary to compose a ten.

1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**Learning Outcomes**

**Operations and Algebraic Thinking**
Use addition and subtraction within 100 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem. (2.OA.1)

Add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two one-digit numbers. (2.OA.2)

**Number and Operations in Base Ten**

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Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens and 6 ones. Understand that 100 can be thought of as a bundle of ten tens — called a “hundred.” (2.NBT.1)

Count within 200; skip count 5’s, 10’s and 100’s. (2.NBT.2)

Read and write numbers to 200 using base-ten numerals, number names, and expanded form (2.NBT.3)

Use place value understanding and properties of operations to add and subtract. Add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.NBT.5)

Explain why addition and subtraction strategies work, using place value and the properties of operations. (2.NBT.9)

**Measurement and Data**

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. (2.MD.6)

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and cent symbols appropriate. *Example: if you have 2 dimes and 3 pennies, how many cents do you have?* (2.MD.8)

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<tr>
<td>2.OA.1</td>
<td>• Recognize that the first digit of a 2-digit number designates the number of groups of 10 and the second digit designates the number of ones</td>
<td>5</td>
<td><strong>Gr. 2 Unit 3- Stickers, Number Strings and Story Problems</strong>&lt;br&gt;Count around the circle by 2s, 5s, and 10s as a warm-up during these lessons.</td>
</tr>
<tr>
<td>2.OA.2</td>
<td>• Solve problems about tens and ones&lt;br&gt;• Add 10 to any number (or any number to 10)</td>
<td></td>
<td><strong>4.1 Groups of 2, 5, and 10</strong>&lt;br&gt;Students will not have done work with even and odd numbers yet. Focus will be on understanding groups of five and ten; not even and odd. Use SAB p. 64 as homework.</td>
</tr>
<tr>
<td>2.NBT.1</td>
<td>• Compose and decompose 2-digit numbers by tens and ones using sticker notation</td>
<td></td>
<td><strong>4.2, Tens and Ones</strong>&lt;br&gt;Use SAB p. 66 as homework if not done in class.</td>
</tr>
</tbody>
</table>
| 2.NBT.2  | • Use a place-value model to represent a number as 10’s and 1’s |      | **4.3 Adding 10**<br>Launch: Introduce Tic Tac Toe Sums from Appendix.<br>This game can be used throughout the year during math workshop to strengthen fluency.  
Workshop: Students should be able to mentally add and subtract 10 (1.OA.5) Use the student activity pages as additional practice.  
Discussion: Focus on how many dimes (tens) there are in 62 cents. |
| 2.NBT.3  | • Find as many combinations of a number as possible, using only 10’s and 1’s |      | **4.4 Sticker Problems**<br>Use SAB p. 74 as homework if not done in class. |
| 2.NBT.5  | Use academic language, such as: even, odd, grouping by 2s, 5s, and 10s, 2-digit number, tens place, ones place |      | **4.5 Making Numbers with Tens and Ones**<br>Use SAB p. 71 for homework.  
Add two additional centers to workshop. Add a center for telling time to quarter hour and money games (Collect and Spend $1.00) |
| 2.NBT.6  | | | |
| 2.NBT.7  | | | |
| 2.NBT.8  | | | |
| 2.MD.7  | | | |
| 2.MD.8  | | | |

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<table>
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<tr>
<th>2.NBT.2</th>
<th>2.NBT.5</th>
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<th>2.NBT.7</th>
<th>2.OA.1</th>
<th>2.MD.7</th>
</tr>
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<tr>
<td>• Add tens and ones to combine 2-digit numbers.</td>
<td>• Use place value, properties of operations and the relationship between operations to solve story problems efficiently.</td>
<td>4</td>
<td>Gr. 2 Unit 6 - How Many Tens? How Many Ones?</td>
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</tr>
<tr>
<td>Use academic language such as: tens, ones, story problems, skip count</td>
<td>2.NBT.2</td>
<td>2.NBT.5</td>
<td>2.NBT.6</td>
<td>2.NBT.7</td>
<td>2.OA.1</td>
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<td>• Make sense or and develop strategies to solve addition and subtraction problems with totals up to 100.</td>
<td>• Understand the equivalence of one group and the discrete units that comprise it.</td>
<td>4</td>
<td>1.1, Story Problems with Stickers</td>
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<td>Use academic language, such as: 100’s chart, hundreds, tens, ones, greater than (&gt;), less than (&lt;), equal to (=), digit, compare, number line</td>
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<td>4</td>
<td>1.2 More Story Problems with Stickers</td>
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</tr>
<tr>
<td>Use academic language, such as: coins, nickel, penny, dime, quarter, cents, array, dollar</td>
<td>2.NBT.2</td>
<td>2.NBT.5</td>
<td>2.NBT.6</td>
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<td>1.4 More Story Problems</td>
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<td>Gr. 2 Unit 6 - How Many Tens? How Many Ones?</td>
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<td>4</td>
<td>2.1 Guess My Number on the 100 Chart</td>
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<tr>
<td>Use academic language, such as: coins, nickel, penny, dime, quarter, cents, array, dollar</td>
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<td>• Make sense or and develop strategies to solve addition and subtraction problems with totals up to 100.</td>
<td>• Understand the equivalence of one group and the discrete units that comprise it.</td>
<td>4</td>
<td>In addition to SAB p. 17, students should complete SAB p. 18</td>
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<tr>
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<td>This game can also be played using a number line.</td>
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<td>2.2 Roll-a-Square</td>
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<td>4</td>
<td>While 2.3 Missing Numbers is not a lesson, you may use the student activity pages for additional work and homework (pp.22-26)</td>
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<tr>
<td>Use academic language, such as: 100’s chart, hundreds, tens, ones, greater than (&gt;), less than (&lt;), equal to (=), digit, compare, number line</td>
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<td>2.4 Sticker Books</td>
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<td>Encourage students to think about equations that represent the problem, as well as the solution, and a way to show the unknown using a blank, a question mark or an empty box. For example, students may suggest 46 + 24 = 70 which is an accurate equation after the problem has been solved, but students should be able to write the following equation 46 + ___ = 70.</td>
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<td>2.5 Strategies for Adding 2-Digit Numbers</td>
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<td>In addition to SAB p. 34, students should complete SAB p. 35A (telling time)</td>
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| 2.NBT.2  | 6 |
| 2.NBT.5  | Grade 2, Unit 6 - How Many Tens? How Many Ones? |
| 2.NBT.6  | Include some Counting Around the Circle by groups of 5 and 10 during the launch or wrap-up. Remember to count back as well as forwards (100,90,80,70...) |
| 2.NBT.7  | You may also counting by adding ten to a given number (7,17,27,37...) |
| 2.OA.1   | For the next four lessons, introduce a new game or activity each day. See Appendix for the following: |

- Make It 4 Ways
- Number Line Explore Game
- Number Compare: Spin to Win
- Mystery Numbers

| 4.1 How Many 5s in 100? |
| 4.2 Skip Counting by 2s, 5s, and 10s |

You may use Make It 4 Ways and Number Line Explore Game during 4.1 and 4.2

Include wrap-up discussions about making connections when skip-counting and working with money.

You may use assessment from 4.3, as additional practice or homework

End-of-Unit Assessment: 1 day
(see Mathematics Department on the Aspen/SIS web page at [https://sis.mybps.org/aspen/index.html](https://sis.mybps.org/aspen/index.html))
UNIT OF STUDY 3: SOLVING STORY PROBLEMS  
November 5-December 2

Primary Curricular Resource:
Grade 2, Unit 3 - Stickers, Number Strings and Story Problems,  
Differentiation and Intervention Guide  
Investigations and the Common Core State Standards  
Time Appendix (see Mathematics Department on the Aspen/SIS web page at https://sis.mybps.org/aspen/index.html )

Estimated Instructional Time: 16 days

Overarching Questions:
- How can students make sense of the action of different addition and subtraction story problems?
- How does understanding the action of a story problem help students determine whether their answer is reasonable?
- How can students show their strategy on a number line?
- How can students use their understanding of place value to help solve problems efficiently?

Instructional Notes:
Consider using Differentiation Guide, Unit 3, pages 34-37  
Number Line: Students should be able to represent their strategies and the action of a story problem on a number line when appropriate (2.MD.6)  
The open number line is a visual representation for recording and sharing students’ thinking strategies during the process of mental computation. Students draw their jumps to explain their thinking process. Before using an empty number line, students need to show a secure understanding of numbers to 100. Prior experiences counting on and back using number lines, recall of addition and subtraction facts for all numbers to ten and the ability to add/subtract a multiple of ten to or from any two-digit number are all important skills that support the efficient use of a number line.

![Number line example]

While students are not responsible for knowing the names of the problem types, Second Grade students need to be proficient with solving for all the addition and subtraction situation types by the end of the year. The practice standards provide an important lens of how to support independent sense-making and active problem-solving.
## Glossary

**Table 1 Common addition and subtraction situations**

<table>
<thead>
<tr>
<th>Add to</th>
<th>Change Unknown</th>
<th>Start Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Result Unknown</strong></td>
<td>Two bunnies were sitting on the grass. Some more bunnies hopped over to the first two? 2 + ? = 5</td>
<td>Some bunnies were sitting on the grass. Three more bunnies hopped there. How many bunnies were on the grass before? ? + 3 = 5</td>
</tr>
<tr>
<td>(K)</td>
<td>(1st)</td>
<td>(2nd)</td>
</tr>
<tr>
<td><strong>Take from</strong></td>
<td>Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 − ? = 3</td>
<td>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? − 2 = 3</td>
</tr>
<tr>
<td>(K)</td>
<td>(1st)</td>
<td>(2nd)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Unknown</th>
<th>Addend Unknown</th>
<th>Both Addends Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Put Together/Take Apart</strong></td>
<td>Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?</td>
<td>Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0</td>
</tr>
<tr>
<td>(K)</td>
<td>(K)</td>
<td>5 = 1 + 4, 5 = 4 + 1</td>
</tr>
<tr>
<td>5 = 2 + 3, 5 = 3 + 2</td>
<td>(1st)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Difference Unknown</th>
<th>Bigger Unknown</th>
<th>Smaller Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compare</strong></td>
<td>(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? 2 + ? = 5, 5 − 2 = ?</td>
<td>(“Version with “more””): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?</td>
</tr>
<tr>
<td>(1st)</td>
<td>(1st)</td>
<td>(1st)</td>
</tr>
<tr>
<td>(Version with “fewer”): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?</td>
<td>(“Version with “fewer””): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?</td>
<td>(1st)</td>
</tr>
<tr>
<td>One-Step Problem</td>
<td>One-Step Problem</td>
<td>One-Step Problem</td>
</tr>
<tr>
<td>(2nd)</td>
<td>(2nd)</td>
<td></td>
</tr>
</tbody>
</table>

**K:** Problem types to be mastered by the end of the Kindergarten year.

**1st:** Problem types to be mastered by the end of the First Grade year, including problem types from the previous year(s). However, First Grade students should have experiences with all 12 problem types.

**2nd:** Problem types to be mastered by the end of the Second Grade year, including problem types from the previous year(s).
Concepts developed in this unit:

Students make sense of mathematics and learn that they can become mathematical thinkers. To this end, students create, use, and share contexts and representations to make sense of problems.

Classroom discussions highlight different ways of interpreting a problem, and solving a problem. Students can use representations to communicate their pertinent mathematical ideas.

Students persevere in solving problems by investigating and practicing problem-solving strategies. They use equations to represent the kind of addition and subtraction situation presented and their solution path.

Prior knowledge expected:

Operations and Algebraic Thinking

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknown in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problems.

1.OA.4 Understand subtraction as an unknown-addend problem. Work with addition and subtraction equations.

1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations: 8 + ? = 11, 5 = __ - 3, 6 + 6 = __.

Learning Outcomes

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction
Use addition and subtraction within 100 to solve one- and two- step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (2.OA)

Numbers and Operations in Base Ten

Use place value understanding and properties of operations to add and subtract.
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.NBT.5)
Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (2.NBT.7)

*Explanations may be supported by drawings or objects.

Measurement and Data

Relate addition and subtraction to length.
Represent whole-number sums and differences within 100 on a number line diagram. (2.MD.6)

Work with time and money.
Tell and write time from analog and digital clocks to quarter hour (2.MD.7)
Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and cent symbols appropriate. Example: if you have 2 dimes and 3 pennies, how many cents do you have? (2.MD.8)
<table>
<thead>
<tr>
<th>MA 2011</th>
<th><strong>After completing each investigation, students will be able to:</strong></th>
<th>Days</th>
<th>Primary Curriculum Resource</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.OA.1</td>
<td>Make sense and solve different types of word problems with the unknown in all positions (result unknown, change unknown, and start unknown).</td>
<td>2</td>
<td>BPS Created Lessons for Visualizing Adding and Subtracting Problems Refer to pp. 15 - 16 in the Differentiation and Intervention Guide During Day 2 - Modeling Story Problems - include number lines as a representation. See Appendix</td>
</tr>
<tr>
<td>2.OA.1</td>
<td>• Make sense and solve different types of word problems with the unknown in all positions (Result unknown, change unknown, and start unknown). • Use place value understanding, properties of operations and the relationship between operations to solve story problems efficiently • Explain with representations and words why a strategy works</td>
<td>7</td>
<td>Gr. 2, Unit 3 - Stickers, Strings, and Story Problems Be sure to model equations using a symbol for the unknown number to represent the problem. For example: $3 + \square = 9$, $16 + 5= \Box$, $20 - \square = 12$</td>
</tr>
<tr>
<td>2.NBT.5</td>
<td>Use academic language such as: count all, add tens and ones, count back, addition, subtraction, equation.</td>
<td>2</td>
<td>2.1 Revisiting Addition Story Problems 2.2 Revisiting Subtraction Story Problems 2.3 Related Story Problems 2.4 Problems with Unknown Change 2.5 Cover Up 2.6 Story Problems 2.7 Assessment: Story Problems You may use this assessment as a mid-unit assessment task.</td>
</tr>
<tr>
<td>2.NBT.5</td>
<td>2.1 &amp; 2.3 - Add To/Result Unknown 2.2 &amp; 2.3 - Take From/Result Unknown 2.4 &amp; 2.5 - Add To/Change Unknown and Take From/Change Unknown 2.6 - All of the above 2.5, 2.6, &amp; 2.7 Include lessons/centers that tell time to the quarter hour somewhere within each workshop. See Time Appendix.</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2.NBT.6</td>
<td>2.1 &amp; 2.3 - Add To/Result Unknown 2.2 &amp; 2.3 - Take From/Result Unknown 2.4 &amp; 2.5 - Add To/Change Unknown and Take From/Change Unknown 2.6 - All of the above 2.5, 2.6, &amp; 2.7 Include lessons/centers that tell time to the quarter hour somewhere within each workshop. See Time Appendix.</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2.NBT.7</td>
<td>2.1 &amp; 2.3 - Add To/Result Unknown 2.2 &amp; 2.3 - Take From/Result Unknown 2.4 &amp; 2.5 - Add To/Change Unknown and Take From/Change Unknown 2.6 - All of the above 2.5, 2.6, &amp; 2.7 Include lessons/centers that tell time to the quarter hour somewhere within each workshop. See Time Appendix.</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2.NBT.9</td>
<td>2.1 &amp; 2.3 - Add To/Result Unknown 2.2 &amp; 2.3 - Take From/Result Unknown 2.4 &amp; 2.5 - Add To/Change Unknown and Take From/Change Unknown 2.6 - All of the above 2.5, 2.6, &amp; 2.7 Include lessons/centers that tell time to the quarter hour somewhere within each workshop. See Time Appendix.</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2.OA.1</td>
<td>• Make sense and solve different types of word problems with the unknown in all positions (start unknown). • Develop strategies for solving addition and subtraction situations with an unknown start. • Use standard notation (+, -, =) to represent addition and subtraction situations with unknown start.</td>
<td>4</td>
<td>Investigations and the Common Core State Standards, Unit 3 - Stickers, Strings, and Story Problems see CC21 Be sure to model equations using a symbol to represent the start unknown. For example: $? + 8 = 12$; $\square + 5 = 15$; $\Box - 7 = 3$.</td>
</tr>
<tr>
<td>2.OA.1</td>
<td>2.5A - Problems with Start Unknown: Add To/Start Unknown 1 day Take From/Start Unknown Problems (1 day) Differentiation and Intervention Guide p. 17 - Story Problems with Multiple Parts (1 day)different positions Additional Word Problems (Add To/Start Unknown with unknowns See Appendix for additional story problems.</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

End of Unit Assessment: 1 day
(see Mathematics Department on the Aspen/SIS web page at https://sis.mybps.org/aspen/index.html)
### UNIT OF STUDY 4: ODD AND EVEN NUMBERS AND STORY PROBLEMS

**Primary Curricular Resource:**
- Gr. 2, Unit 3 - *Stickers, Number Strings and Story Problems and Investigations*
- Gr. 2, Unit 8 - *Partners, Teams and Paper Clips*

*Investigations and the Common Core State Standards Guide*

Unit of Study 4 Appendix (see Mathematics Department on the Aspen/SIS web page at [https://sis.mybps.org/aspen/index.html](https://sis.mybps.org/aspen/index.html))

**Estimated Instructional Time:** 27 days

**Overarching Questions:**
- How do students apply what they know about place value and the properties of operations to help solve story problems?
- What generalizations can students make about odd and even numbers?
- How do students represent and notate efficient addition (keeping one number whole, adding by place value) and subtraction strategies (adding up or subtracting back in parts)?

**Instructional Notes:**

**Fluency:**

“The word *fluent* is used in the Standards to mean “fast and accurate.” Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., “adding 0 yields the same number”), and knowing some answers from the use of strategies... So the important press toward fluency should also allow students to fall back on earlier strategies when needed. By the end of the K–2 grade span, students have sufficient experience with addition and subtraction to know single-digit sums from memory; 2.OA.2 as should be clear from the foregoing, this is not a matter of instilling facts divorced from their meanings, but rather as an outcome of a multi-year process that relies heavily involves the interplay between reasoning and practice.”

*-OA Progression pages 18-19*

The mental strategies referred to in 2.OA.2 are listed in the first grade standard 1.OA.6. While counting on is a early strategy, students need to use additional mental strategies to develop fluency with single digit combinations:

- making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14)
- decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9)
- using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4)
- creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13)

For additional information: [http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf](http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf)
**Two-Step Problems:** Most two-step problems made from two easy subtypes are easy to represent with an equation, as shown in the first two examples below. But problems involving a comparison or two middle difficulty subtypes may be difficult to represent with a single equation and may be better represented by successive drawings or some combination of a diagram for one step and an equation for the other (see the first three examples). Students can make up any kinds of two-step problems and share them for solving (*Progressions*). (If you have students that finish solving two-step story problems, you may ask them to write their own two-step story problems and have their classmates solve them.)

![Examples of two-step Grade 2 word problems](image)

<table>
<thead>
<tr>
<th>Two easy subtypes with the same operation, resulting in problems represented as, for example, $9 + 5 = 14$ or $16 - 8 = 8$ and perhaps by drawings showing these steps:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example for $9 + 5$: There were 9 blue balls and 5 red balls in the bag. Ask put in 7 more balls. How many balls are in the bag altogether?</td>
</tr>
<tr>
<td>Two easy subtypes with opposite operations, resulting in problems represented as, for example, $9 - 5 = 4$ or $16 - 8 = 8$ and perhaps by drawings showing these steps:</td>
</tr>
<tr>
<td>Example for $9 - 5$: There were 9 carrots on the plate. The girls ate 5 carrots. Mother put 7 more carrots on the plate. How many carrots are there now?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>One easy and one middle difficulty subtype:</th>
</tr>
</thead>
<tbody>
<tr>
<td>For example: Maria has 9 apples. Corey has 4 fewer apples than Maria. How many apples do they have in all?</td>
</tr>
<tr>
<td>For example: The zoo had 7 cows and some horses in the big pen. There were 15 animals in the big pen. Then 4 more horses ran into the big pen. How many horses are there now?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Two middle difficulty subtypes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>For example: There were 9 boys and some girls in the park. In all, 15 children were in the park. Then some more girls came. Now there are 14 girls in the park. How many more girls came to the park?</td>
</tr>
</tbody>
</table>

**Number Line:**
Continue to use number line as a tool to represent certain strategies.
Possible questions include:
- We can we go from 27 to 53 in a small number of jumps? Who has another way?
- How can we go from 62 to 45 in a small number of jumps? Who has another way?
- How can we solve $34 + 23$? (counting on without crossing the tens boundary)
- How can we solve $37 + 25$? (counting on crossing the tens boundary)
- How can we solve $47 - 23$? (counting back without crossing the tens boundary)
- How can we solve $42 - 27$? (counting back crossing the tens boundary)
- How can we solve $82 - 47$? (counting up)

*(from North Carolina Unpacking Standards)*

**Please note:** The lessons from *the Investigations and the Common Core State Standards* for Unit 8 will be used as part of Unit of Study 6. Do **not** use these lessons as part of this unit.

**Unit 3 – Stickers, Strings and Story Problems** includes only lessons from Investigation 3 (3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7).

**Unit 8 – Partners, Teams and Paper Clips** After 3.5 there will be one day of BPS: 2-step story problems (located on Elementary Math page)

**Time:** Be sure to incorporate *telling time to the quarter hour* during math workshop several times during this unit. **See Time Appendix.**
Concepts developed in this unit:
Students work with equal groups of objects to gain foundations for multiplication and represent (number lines and base ten models) and solve problems involving addition and subtraction.

Students will characterize even and odd numbers as those that do or do not make groups of two (partners) and two equal groups (teams). Students should explore this concept with concrete objects (e.g., counters, cubes, etc.) before moving towards pictorial representations such as circles or arrays.

Students will also consider the relationship between skip counting and grouping as preparation for the work they will do with multiplication in grade 3.

Prior knowledge expected:
Measurement and Data
MA.5. Identify the values of all U.S. coins and know their comparative values (e.g. a dime is of greater value than a nickel). Find equivalent values (a nickel is equivalent to 5 pennies). Use appropriate symbols. Use the values of coins in the solutions of problems.

Operations and Algebraic Thinking
1.OA.3 Apply properties of operations as strategies to add and subtract.
1.OA.4 Understand subtraction as an unknown-addend problem. 
Add and Subtract within 20.
1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use mental strategies such as counting on, making ten, decomposing a number leading to a ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums.
Work with addition and subtraction equations.
1.OA.8 Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers.

Numbers and Operations in Base Ten
1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. 

Operations and Algebraic Thinking
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (2.OA.1)
Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. (2.OA.3)
Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends (2.OA.4)

Number and Operations in Base Ten
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.NBT.5)

Measurement and Data
Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram (2.MD.6)
Tell and write time from analog and digital clocks to the nearest hour. (2.MD.7)
Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and cent symbol appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? (2.MD.8)
<table>
<thead>
<tr>
<th>MA 2011</th>
<th>After completing each investigation, students will be able to:</th>
<th>Days</th>
<th>Primary Curriculum Resource</th>
</tr>
</thead>
</table>
| 2.OA.3  | Determine whether a group of objects has an odd or even number of members. Write an equation to express an even number as a sum of two equal addends. | 7    | Unit 3 – *Stickers, Strings and Story Problems*  
ONLY Investigation 3:  
3.1, Partners and Teams  
3.2, More Partners and Teams  
3.3, Assessment: Even or Odd?  
3.4, How Many Fingers in Our Class?  
3.5, Collect 50¢  
3.6, Counting by Groups  
3.7, More Counting by Groups  
3.6 & 3.7 – Include activities about telling time to the quarter hour during Math Workshop. See Time Appendix. |
| 2.MD.7  | Characterize even and odd numbers as those that do or do not make groups of two (partners) and two equal groups (teams.)  
Investigate what happens with partners and teams when two groups are combined.  
Make and justify generalizations about adding even and odd numbers. | 4    | Unit 8 – *Partners, Teams and Paper Clips*  
1.1 Partners and Teams with Two Groups  
1.2 More Partners and Teams with Two Groups  
1.3 Adding Even and Odd Numbers  
1.4 More Adding Even and Odd Numbers  
See Appendix: Add the game “Odd and Even” during math workshop. |
| 2.OA.2  | Relate unknown combinations to known combinations.  
Develop fluency with the plus 9 and remaining combinations.  
Use cubes and the number line to show how addition combinations are related. | 2    | Unit 8 – *Partners, Teams and Paper Clips*,  
2.1 Plus 9 or 10 BINGO  
2.2 The Remaining Combinations  
2.1 & 2.2 – While students played Plus Ten Bingo in first grade, the game Plus 9 or 10 Bingo provides opportunities for students to build fluency and develop their mental math strategies. Students should understand the relationship between adding 10 and adding 9 to solve plus 9 problems. |
| 2.OA.1  | Make sense of and develop strategies to solve addition and subtraction problems to 100.  
Use addition and subtraction to solve 2-step problems using various story problem situations.  
Solving subtraction problems by subtracting in parts, by adding up, or subtracting back to find the difference.  
Compare problems in which the amount subtracted differs by 1. | 7    | Unit 8 – *Partners, Teams and Paper Clips*, 3.1 - 3.5 and BPS lesson  
3.1 Pinching Paper Clips  
3.2 Subtracting in Parts  
3.3 Adding Up or Subtracting Back  
3.4 Story Problems  
3.5 Assessment: Paper Clips and Cherries  
3.3, 3.4 & 3.5 – Include telling time during Math Workshop. This should be based on student need (telling time to the hour, half hour, quarter hour, every 5 minutes) |
| 2.NBT.5 |                                                                                     |      |                                                                             |
| 2.NBT.9 |                                                                                     |      |                                                                             |

**Boston Public Schools Elementary Mathematics Department**  
**Grade 2 Scope and Sequence, 2015-2016. Last updated 8.12.15**

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<table>
<thead>
<tr>
<th>Use academic language, such as:</th>
<th>Two-Step Word Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>visualize, retell, model, open number line, generalization, notation, equal groups</td>
<td>After 3.5 there will be 2 days of BPS: 2-step word problems. See Appendix</td>
</tr>
</tbody>
</table>

2.NBT.5 2.NBT.9
- Add, make and justify generalizations about adding even and odd numbers.
- Make sense of solve addition and subtraction problems with totals to 100, develop efficient strategies, and methods for adding and notating strategies.
- Add 2-digit numbers by keeping on number whole, and adding by tens and ones.
- Notice what happens to place value when 2-digit numbers with a sum over 100 are combined.
- Use sticker notation and the number line to show strategies and solutions.

Use academic language, such as:
two-digit numbers, generalizations, standard notation.

<table>
<thead>
<tr>
<th>4</th>
<th>Unit 8 – Partners, Teams and Paper Clips</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Strategies for Addition</td>
<td>Most students begin with $40 + 30$ or $30 + 40$. Their second steps will vary (e.g. $8 + 3 = 11, 70 + 8 = 78, or 70 + 3 = 73$), as will the ways they carry out these problem parts (e.g., adding 11 as a whole vs. breaking it into 10 and 1). Focus students’ attention on the first step ($40 + 30$) to help them see what is the same in each of these methods and to understand why they are all considered “adding tens and one.”</td>
</tr>
<tr>
<td>4.2 Keeping One Number Whole</td>
<td>The student activity pages show both equations and vertical notations for addition. If a student uses the standard algorithm to solve, ask him or her to solve using another strategy.</td>
</tr>
<tr>
<td>4.3 Adding Tens and Ones</td>
<td>4.4 Adding 2-Digit Numbers</td>
</tr>
<tr>
<td>4.5 End-of-Unit Assessments</td>
<td></td>
</tr>
</tbody>
</table>

End-of-Unit Assessment: 1 day
(see Mathematics Department on the Aspen/SIS web page at [https://sis.mybps.org/aspen/index.html](https://sis.mybps.org/aspen/index.html))
### Unit of Study 5: Understanding Place Value 2
#### January 22-March 18

**Primary Curricular Resource:**
Gr. 2, *Investigations and the Common Core Standards* (Unit 6 - 5.1A - 5.5A),  
Gr. 3, Unit 1 - *Trading Stickers and Combining Coins* (Investigation 1) and  
Gr. 2, *Investigations and the Common Core Standards* (Unit 8 – 5.1A – 5.5A)  
Unit of Study 5 Appendix (see Mathematics Department on the Aspen/SIS web page at [https://sis.mybps.org/aspen/index.html](https://sis.mybps.org/aspen/index.html)

**Estimated Instructional Time:** 34 days

**Overarching Questions:**
- How does using place value and the properties of operations help students develop efficient addition and subtraction strategies?
- How many tens are there in a given 3-digit number?
- How do representations like the base-ten model, number line, sticker station model, etc. strengthen and show understanding of the properties of place value?

**Instructional Notes:**

#### Place Value:
Students extend their understanding of the base-ten system by exploring the idea that numbers such as 100, 200, 300, etc., are groups of hundreds with zero tens and ones. Students can represent this with both groupable (cubes, links) and pre-grouped (place value blocks) materials.

*As in First Grade, Second Graders’ understanding about hundreds also moves through several stages:*

**Counting By Ones:** At first, even though Second Graders will have grouped objects into hundreds, tens and left-overs, they rely on counting all of the individual cubes by ones to determine the final amount. It is seen as the only way to determine how many.

**Counting By Groups and Singles:** While students are able to group objects into collections of hundreds, tens and ones and now tell how many groups of hundreds, tens and left-overs there are, they still rely on counting by ones to determine the final amount. They are unable to use the groups and left-overs to determine how many.

**Counting by Hundreds, Tens & Ones:** Students are able to group objects into hundreds, tens and ones, tell how many groups and left-overs there are, and now use that information to tell how many. Occasionally, until this stage becomes fully developed, second graders rely on counting to “really” know the amount, even though they may have just counted the total by groups and left-overs.

Understanding the value of the digits is more than telling the number of tens or hundreds. Second Grade students who truly understand the position and place value of the digits are also able to confidently model the number with some type of visual representation. Others who seem like they know, because they can state which number is in the tens place, may not truly know what each digit represents.

> *- From North Carolina Unpacking Standards, Grade 2*

*Please be aware that when reading and writing whole numbers the word “and” should not be used (e.g., 235 is stated and written as “two hundred thirty-five”).*
Applying understanding of place value to adding and subtracting:
It is important to highlight the relationship between addition and subtraction throughout this unit. Expanded form (125 can be written as 100 + 20 + 5) is a valuable skill when students use place value strategies to add and subtract large numbers in 2.NBT. 7.
When some students solve a problem by subtracting and others solve the same problem by finding the missing addend, take the opportunity to comment on the relationship between addition and subtraction.
There are various strategies that students understand and use when adding and subtracting within 100 (such as those listed in the standard).

The standard algorithm of carrying or borrowing is neither an expectation nor a focus in Second Grade. Students use multiple strategies for addition and subtraction in Grades K-3. By the end of Third Grade students use a range of algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction to fluently add and subtract within 1000. Students are expected to fluently add and subtract multi-digit whole numbers using the standard algorithm by the end of Grade 4.

Second graders apply their place value understanding to two 3-digit numbers. Students should have ample experiences using concrete materials and pictorial representations to support their work.

This standard also references composing and decomposing a ten or a hundred. This work should include strategies for solving a computational problem such as making a 10, making a 100, breaking apart a 10, or creating an easier problem. The standard algorithm of carrying or borrowing is not an expectation in Second Grade.

**Number Line:** The number line can be a tool to illustrate how subtraction is finding the difference between two numbers. For example in 243-87, one can add up from 87 (the subtrahend) to the get to 243 (the minuend). In this way, students can see 243-87= ? is the same as 87 + ? = 243.

To use a number line diagram to understand number and number operations, students need to understand that number line diagrams have specific conventions: the use of a single position to represent a whole number and the use of marks to indicate those positions. They need to understand that a number line diagram is like a ruler in that consecutive whole numbers are 1 unit apart, thus they need to consider the distances between positions and segments when identifying missing numbers. These understandings underlie students; successful use of number line diagrams.

The ability to mentally add and subtract multiples of 10 to a given number 100-900 is a grade 2 standard (2.NBT.8)
During this unit observe students informally to assess their progress they are making towards meeting this standard. This standard is assessed in Sessions 1.6 and 1.7 during Math Workshop. A checklist is provided to record your observations.
Skip Counting: Second grade students also begin to work towards multiplication concepts as they skip count by 5s, by 10s, and by 100s.

If you count by 5’s, and start at 27, what other numbers will be in the pattern?
If you start at 438 and count by 5s and then start at 438 and count by 10s, what are three numbers that will come up in each pattern? (10s: 438, 448, 458, 468, 478) (5s: 438, 443, 448, 453, 458, 463, 468, 473, 478)
Starting at 100, what are all the numbers you can skip count by to get to 150? Give examples to support your answer.
If you start at 17 and count by 10s, will you land on 100? Why or why not?
What patterns do you see in the ones, tens, and hundreds place when skip counting by 5s? 10s? 100s?

Summer started on 205. She counted by 100s. Is 808 in her pattern? Explain how you know.

From https://grade2commoncoremath.wikispaces.hcpss.org/2.NBT.2

Additional Resources:
https://grade2commoncoremath.wikispaces.hcpss.org/Skip+Counting+Activities
https://grade2commoncoremath.wikispaces.hcpss.org/2.NBT.2

Gr.2, Investigations and the Common Core State Standards
- 5A.3 & 5A.4 – You may choose to add an additional center to your Math workshop that focuses on students telling time to the nearest 5 minutes. This is the standard by the end of Grade 2.

Gr. 3, Unit 1 - Trading Stickers, Combining Coins
- 1.1 - Omit Activity 3 - Introducing What’s the Temperature
- 1.6 - Also have students play Spend $2.00 (Refer to spend $1.00 in Unit 6 for directions. You will need a 200 chart)
- Omit 2.1 (taught at the beginning of the year)
- 2.5 Omit - Omit Activity 1A - Assessment: Addition Combinations

Concepts developed in this unit:
Students extend their understanding of the base-ten system. This includes counting multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing.

Students understand multi-digit numbers (up to 1000) written in base-ten notations, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

Students use their understanding of addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, Prior knowledge expected: Number and Operations in Base Ten

1.NBT.2 Understand that the two-digits of a two-digit number represent amounts of tens and ones. Understand the following special cases:
   a. 10 can be thought of as a bundle of ten ones - called a “ten”
   b. The numbers 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones).

1.NBT.3 Compare two two-digit numbers based on the meanings of the tens and ones digit, recording the results of comparison with the symbols >, <, or =.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens and ones and ones; and sometimes it is necessary to compose a ten.
| accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. | 1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.  
1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. |

**Student Learning Outcomes**

**Number and Operations in Base Ten**

*Understand Place Value.*

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 one. Understand the following as special cases:

- a. 100 can be thought of as a bundle of ten tens - called “a hundred.”
- b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (2.NBT.1)

**Count within 1000; skip-count by 5s, 10s, and 100s.** (2.NBT.2)

**Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.** (2.NBT.3)

**Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, < symbols to record the results of comparisons.** (2.NBT.4)

**Use place value understanding and properties of operations to add and subtract.**

- Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.NBT.5)
- Add up to four two-digit numbers using strategies based on place value and properties of operations. (2.NBT.6)
- Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens and hundreds. (2.NBT.7)
- Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900 (2.NBT.8)

**Measurement and Data**

*Relate addition and subtraction to length.*

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, . . . and represent whole-number sums and differences within 100 on a number line diagram. (2.MD.6)
<table>
<thead>
<tr>
<th>MA 2011</th>
<th>After completing each investigation, students will be able to:</th>
<th>Days</th>
<th>Primary Curriculum Resource</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.NBT.1 2.NBT.2 2.NBT.3 2.NBT.4 2.NBT.5 2.NBT.6 2.NBT.7 2.NBT.8 2.NBT.9</td>
<td>• Add and subtract 10 or 100 from a given number and describe what part of the number changes  • Use &lt; and &gt; notation to compare numbers  • Read and write 3-digit numbers  • Use a place-value model to represent and compare 3-digit numbers as 100s, 10s, and 1  • Represent 2- and 3-digit numbers using expanded form  • Recognize that the numbers 100, 200, 300 represent groups of 100  • Identify the value that each digit in a 3-digit number represents  • Represent whole-number sums and differences on a number line diagram</td>
<td>5</td>
<td>Investigations and the Common Core State Standards  Unit 6  5A.1 Numbers on the 200 Chart  5A.2 Stickers: How Many Hundreds, Tens, and Ones?  5A.3 How Many Hundreds?  5A.4 Adding and Subtracting 10 and 100  5A.5 Assessment: Hundreds, Tens and Ones  5A.3 &amp; 5A.4 – You may choose to add an additional center to your math workshop that focuses on money and/or time. (See Time and Money appendices)</td>
</tr>
<tr>
<td>2.MD.6 2.MD.7</td>
<td>Use academic language, such as: 1-digit number, 2-digit number, 3-digit number, open number line, place value, digit, hundreds, tens, ones, identify, place value model, expanded for/notation, clock times to the nearest 5 minutes.</td>
<td>8</td>
<td>Grade 3 Unit 1 – Trading Stickers and Combining Coins  1.2 Adding and Subtracting 2-Digit Numbers See Appendix for Get to Zero. Introduce Get to Zero as an additional game.  1.3 More Than Ten Ones  1.4 How Many More Stickers to Get 100?  1.5 Capture 5: Adding and Subtracting 10s and 1s  1.6 Assessment: Adding and Subtracting 10s As part of math workshop you may choose to also play Spend $2.00 and/or a station for telling time to every 5 minutes  1.7 Strategies for Capture 5  1.8 Making Numbers with 100s, 10s, and 1s  1.9 Assessment: Hundreds, Tens, and Ones</td>
</tr>
<tr>
<td>2.NBT.1 2.NBT.3 2.NBT.4 2.NBT.5 2.NBT.6 2.NBT.7 2.NBT.8 2.NBT.9 2.MD.6 2.MD.7 2.MD.8</td>
<td>• Use place value understanding to compose and decompose numbers  • Recognize and represent the place value of each digit in a 2- and 3-digit number.  • Find different combinations of a number using 100’s, 10’s, and 1’s and recognizing their equivalence.  • Read and write numbers from standard notation to expanded form and from expanded form to standard notation.  • Use the structure of the base ten number system up to 1,000 to solve problems and develop efficient strategies to solve up to 3-digit addition and subtraction problems.  • Generalize about numbers and operations.  Use academic language, such as: digit, addition, subtraction, equation, expression, tens place, ones place, difference, open number-line, adding one number in parts, adding by place, penny, nickel, dime, quarter, dollar, sheet, strip, single</td>
<td></td>
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</tbody>
</table>

Boston Public Schools Elementary Mathematics Department  
Grade 2 Scope and Sequence, 2015-2016. Last updated 8.12.15
### Grade 2 Scope and Sequence

**2.NBT.1**
- Represent 3-digit numbers using a place-value model.
- Add two 3-digit numbers by combining hundreds, tens, and ones.
- Solve a subtraction problem by keeping one number whole and subtracting the other parts by place.
- Subtract numbers where it is necessary to regroup the number of tens (or hundreds) in the total amount.
- Make and justify generalizations about adding even and odd numbers.
- Develop and achieve fluency with the Plus 9 and remaining combinations.
- Add 2- digit and 3-digit numbers accurately and efficiently.

**Use academic language, such as:**
- hundreds, tens, ones, 3-digit number, sheets, strips, singles, place value, subtracting by place, keeping one number whole – subtracting the other number in parts, sticker notation

### Grade 3 Unit 1 – Trading Stickers and Combining Coins

**2.2 Close to 100**

**2.3 More or Less Than 100?**

**2.4 Coin Combination**

**2.5 Assessment: Addition Combinations (to 20)**

**Omit Activity 1.A**

*See Appendix for Close to 100 with 3 Numbers.* Add this game to the math workshop. May also add additional center on telling time to the quarter hour.

**2.6 Story Problem Strategies**

**2.7 163 Stickers**

Add *Close to 100 with 3 Numbers* game to math workshop.

**2.1 Addition Combinations:** This lesson can be used as an addition resource for students who need practice with mental strategies and fluency with basic combinations.

**2.8 End-of-Unit Assessment**

You may use some or all of this problems as an additional resource. By the end of this session, students should be fluent with adding and subtracting two-digit numbers. *See Appendix for additional 2-digit subtraction problems.*

**Investigations and the Common Core Standards** book, Unit 8

**5A.1 Combining Stickers**

**5A.2 Adding Hundreds, Tens, and Ones**

**5A.3 Subtracting Hundreds, Tens, and Ones**

**5A.4 Subtracting 3-Digit Numbers**

**5A.3 –** The work in these sessions focuses on the subtraction strategy of keeping one number whole and then subtracting the amount being removed by breaking it into hundreds, tens, and ones and subtracting each of those amounts. This strategy is well represented using the familiar context of stickers and place value notation. Also encourage students to use other subtraction strategies (including adding up to find the difference) and representing these strategies on an open number line.

**5A.3 & 5A.4 –** Students should be using the open number line to represent both addition and subtraction of large numbers. If students are using sticker notation for subtracting 3-digit numbers this should be paired with equations that represent each part being removed.
| 2.NBT.4 | • Create and Compare 3 Digit Numbers |
| 2.NBT.5 | • Mentally add multiples of 10 and 100 to given numbers |
| 2.NBT.6 | • Compose multiples of 100 to make 1000 |
| 2.NBT.7 | • Solve addition problems within 1000 |
| | **Use academic language, such as:** |
| | thousand, hundreds, tens, ones, 3-digit number, place value, a.m. and p.m. |

| 6 | See **Unit of Study 5 Appendix** for the following lessons and additional resources: |
| | **My 1000 Book** |
| | **Place Value Challenge** |
| | Introduce Game during Launch. |
| | During Math Workshop, other related games can be included, such as Close to 100 as well as working on the 1000 book. |
| | You can include Problem #6 from Unit 8, 5A.5 during this lesson. |
| | **What Number Is...?** |
| | Use time and money activities during math workshop. |
| | **3 days:** |
| | Adding and Subtracting 3-digit Numbers |
| | Money and Time Activities |

| **End-of-Unit Assessment:** 1 day |
| (see Mathematics Department on the Aspen/SIS web page at [https://sis.mybps.org/aspen/index.html](https://sis.mybps.org/aspen/index.html)) |
UNIT OF STUDY 6: GEOMETRY AND EQUAL SHARES  
March 21 – April 28

Primary Curricular Resource:  
Grade 2, Unit 2 - Shapes, Blocks, and Symmetry  
Gr. 2, Unit 7 - Parts of a Whole, Parts of a Group  
Unit of Study 6 Appendix (see Mathematics Department on the Aspen/SIS web page at https://sis.mybps.org/aspen/index.html)

Estimated Instructional Time: 25 days

Overarching Questions:
- What are the attributes (features, characteristics) of 2-dimensional and 3-dimensional shapes?  
- How can shapes be composed and decomposed in different ways to sort, categorize, name and think about the relationships between them?  
- How do students find the area of a rectangle?  
- How can students partition rectangles and circles into halves, thirds and fourths?  
- How can identical wholes have equal shares that are different shapes (halves, thirds, fourths)?

Instructional Notes:

Geometry:
Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Partitioning Rectangles into Rows and Columns
Second graders use rectangular arrays to work with repeated addition, a building block for multiplication in third grade. A rectangular array is any arrangement of things in rows and columns, such as a rectangle of square tiles. Students explore this concept with concrete objects (e.g., counters, bears, square tiles, etc.) as well as pictorial representations on grid paper or other drawings. Due to the commutative property of multiplication, students can add either the rows or the columns and still arrive at the same solution.

This standard relates to 2.OA.4: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. The lessons are designed to show the coherence between the two standards.

Fractions as a Area Model
Second Grade students partition circles and rectangles into 2, 3, or 4 equal shares (regions). Students should be given ample experience to explore this concept with paper strips and pictorial representations. Students should also work with the vocabulary terms halves, thirds, half of, third of, and fourth (or quarter) of. While students are working on this standard, teachers should help them to make the connection that a “whole” is composed of two halves, three thirds, or four fourths.

- Throughout the fraction part of this unit make sure to make connections to telling time to the quarter hour and fourths of a circle.

- Ask questions related to the idea that equal shares of identical wholes may not have the same shape. Example:
**Teacher:** Can you partition each rectangle into fourths a different way?  
**Student A:** I partitioned this rectangle 3 different ways. I folded or cut the paper to make sure that all of the parts were the same size.

![Partitioned rectangles](image)

**Teacher:** In your 3 pictures, how do you know that each part is a fourth?  
**Student:** There are four equal parts. Therefore, each part is one-fourth of the whole piece of paper.

It is important for students to understand that fractional parts may not be symmetrical. The only criteria for equivalent fractions are that the area is equal, as illustrated in the first example above.

Encourage discuss connections between fourths and quarters (money) when appropriate.

**Unit 2 – Shapes, Blocks and Symmetry**

- 1.4 – Omit Activity 1 – Addition Combinations: Doubles, Do Activity 2 and 3
- 1.5 – Omit
- 2.5 – Omit – use of geoboards are optional during this unit
- 2.8 & 2.9 – Omit
- Investigation 3 – Omit (symmetry is a grade 3 standard)
- Use **Differentiation Guide** pages 18-25

**Unit 7 – Parts of a Whole, Parts of a Group:** OMIT any activity or homework related to fractions of a set

- Combine 1.1 and 1.3 (Only do Half of Objects, not Half of a Set)
- 1.2 – Omit (Fractions of a set is not a grade 2 standard)
- 1.4 – Do Activity 1 and 3, Omit Activity 2B – Sharing a picnic
- 2.3 – When discussing flags 2 and 3, point out that 4/4 is also a fraction. It is a way of notating that four of the four equal parts are shaded and that 4/4 is another way of saying one whole or 1.
- 2.3A – Activity 1 – Halves of a Circle will follow the combined 1.1 and 1.3 lesson. Activity 2 – Thirds and Fourths of a Circle will follow 2.4
- 2.5 – OMIT (because this is fractions of a set) and 2.6 – Embedded assessment: eliminate problem 2

Use **Differentiation Guide** pages 78-85

Additional Resource:  
### Concepts developed in this unit:

Students describe and analyze shapes by examining their sides and angles. Students investigate, describe and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing, two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity and symmetry in later grades.

Students can partition circles and rectangles into 2, 3, or 4 equal shares and reason about the relationship between the number of equal shares and the size of each share.

### Prior knowledge expected:

**Geometry**

*Reason with shapes and their attributes.*

1.G.1 Distinguish between defining attributes versus non-defining attributes; build and draw shapes that possess defining attributes.

1.G.2 Compose two-dimensional shapes and three-dimensional shapes to create a composite shape, and compose new shapes from the composite shape.

1.G.3 Partition circles and rectangles into tow and four equal shares, describe the shares using the words *halves, fourths,* and *quarters,* and use the phrases *half of, fourth of,* and *quarter of.* Describe the whole as two of, or *four of* shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

### Learning Outcomes:

**Operations and Algebraic Thinking**

*Work with equal groups of objects to gain foundations for multiplication.*

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (2.OA.4)

**Geometry**

*Reason with shapes and their attributes.*

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons and cubes. (2.G.1)

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (2.G.2.)

Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. (2.G.3)

**Measurement and Data**

*Work with time and money.*

Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (2.MD.7)

Know the relationships of time, including seconds in a minute, minutes in an hour, hours in a day, days in a week, a month, and a year; and weeks in a month and a year. (MA.7.a)
<table>
<thead>
<tr>
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</tr>
</thead>
</table>
| 2.G.1   | • Name the attributes of 2D and 3D shapes and distinguish between them.  
• Compose and decompose shapes in different ways.  
• Sort, categorize, name and think about the relationships between shapes.  
**Use academic language, such as:** polygon, geometry, two-dimensional shape, three-dimensional shape, face. | 4    | Unit 2, *Shapes, Blocks and Symmetry*  
1.1 Geoblock Faces  
Use Student Math Handbook, p.117, Naming Polygons as a reference.  
1.2 Working with Geoblocks  
1.3 Sorting Geoblocks  
1.4 Addition Combinations and Ways to Fill  
Omit Activity 1 – Addition Combinations Doubles, Do Activity 2 and Activity 3 |
| 2.G.1   | • Name the attributes of quadrilaterals and rectangles.  
• Compose and decompose shapes in different ways.  
• Understand that rectangles can be filled with square tiles to find the area.  
**Use academic language, such as:** angle, right angle, rectangle, quadrilateral, area. array | 8    | 2.1 Assessment: Sorting Shapes by Number of Sides  
Notes from *Investigations and CCSS:*  
**Sorting and Naming Polygons:** Polygons are named for the number of sides they have. Students are likely to be familiar with the regular pentagon (Shape U) but be surprised that Shape T is also a pentagon because it has 5 sides. Similarly, they will be surprised by the hexagons Shape V (regular) and Shape I and the octagons Shape X (regular) and Shape W. Look for opportunities throughout Sessions 2.1 and 2.2 to point out and discuss pentagons and hexagons.  
Discuss the name of each shape and the number of sides and angle.  
2.2 Sorting Quadrilaterals  
2.3 Ordering Quadrilaterals  
2.4 Building Rectangles  
**Include Double It in Math Workshop**  
See Inv. And CCSS, page C14 (Double It recording Sheet)  
2.6 How Many Rectangles?  
2.7 Assessment: Is It a Rectangle  
1 Day: Making Rectangular Arrays  
**See Appendix** for  
Building Arrays and Array Memory Game  
Omit 2.5, 2.8, 2.9  
2.10A End of Assessment and Fly Away  
You may choose to use only questions 1 and 2 of the assessment, as most students should already be fluent with double combinations and subtraction facts.  
Use array games from Appendix during Math Workshop. |
| 2.G.3 2.MD.7 2.MD.8 | • Understand that an equal share of a whole does not need to have the same shape.  
• Use fraction notation to label equal parts.  
• Recognize that equal shares of identical wholes need not have the same shape.  
• Partition rectangles into equal parts to show halves, thirds, and fourths  

**Use academic language, such as:**  
one half, fraction, equal, half, thirds, fourths, semi-circle, one-fourth, one-quarter, one-third, two-thirds, three-fourth, two-halves, three-thirds, four-fourths  

Tell and write time from analog and digital clocks to the nearest 5 minutes, using a.m. and p.m. | 10 | **Unit 7, Parts of a Whole, Parts of a Group**  
**1.1 What Is a Half? and 1.3 Halves of Blocks and Balloon Bunches**  
Combine 1.1 and 1.3  
Only do fractions of a half and not of a set. Do not do activities related to finding half of a geo-block.  

**Omit 1.2 & 1.4**  

**2.3A Parts of Circles**  
2.3A – Activity 1 – Halves of Circles (1 day)  
Activity 2 – Fourths and Thirds of Circles. Make a connection to telling time to the quarter hour and finding fourths of a circle, and quarters in money. (1 day)  

**Day 3 Telling Time (to the nearest 5 minutes). See Appendix**  

**Day 4 Task from Illustrative Mathematics – Representing Half of a Rectangle and Which Pictures Represent One-Half? See Appendix.**  

**2.1 Fourths of a Square**  
**2.2 Thirds of a Flag**  
**2.3 More Fraction Flags**  
**2.4 Fraction Flag Poster**  
Omit 2.5  
**2.6 – Embedded assessment – omit problem 2**  

During these sessions, have students practice telling time to the nearest 5 minutes.  

**Day 10: Finding Equivalent Wholes (e.g. /4 = 1)**  

End-of-Unit Assessment: 1 day  
(see Mathematics Department on the Aspen/SIS web page at [https://sis.mybps.org/aspen/index.html](https://sis.mybps.org/aspen/index.html))
### UNIT OF STUDY 7: MEASUREMENT AND DATA
April 29 – June 22

**Primary Curricular Resource:**
Gr. 2, Unit 9 - *Measuring Length and Time* (Inv. 1, 2, 3)  
Gr. 2, Unit 4 - *Pockets, Teeth and Favorite Things* (Inv. 1)  
*Investigations and the Common Core State Standards – Unit 9*

Unit of Study 7 Appendix (see Mathematics Department on the Aspen/SIS web page at [https://sis.mybps.org/aspen/index.html](https://sis.mybps.org/aspen/index.html))

**Estimated Instructional Time:** 29 days

<table>
<thead>
<tr>
<th>Overarching Questions:</th>
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<tbody>
<tr>
<td>• What do we need to understand in order to measure accurately?</td>
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<tr>
<td>• Why is it important to have a standard unit of measurement?</td>
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<tr>
<td>• How can a line plot be a tool for organizing data?</td>
</tr>
<tr>
<td>• How can we describe the information in a bar graph or picture graph?</td>
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</table>

**Instructional Notes:**

**Non-Standard and Standard Measurement:**
As students experience and complete tasks that involve real measurement, encourage discussions that highlight key measurement ideas:

1.) larger units (e.g. yard) can be subdivided into equivalent units (e.g. inches) (partition),  
2.) the same object or many objects of the same size such as paper clips can be repeatedly used to determine the length of an object (iteration),  
3.) The smaller the unit, the more units it will take to measure the selected attribute of an object (compensatory principal).

Provide students with opportunities to create and use a variety of rulers so that they can connect their understanding of non-standard units from first grade to standard units in second grade. As you begin this unit, help students find connections to the work of grade 1.

As students create a ruler with evenly spaced points corresponding to the numbers, ask questions to encourage them to recognize the similarities between a number line and a ruler. (2.MD.6)

**Categorical Data:**
In Second Grade, students pose a question, determine up to 4 categories of possible responses, collect data, represent data on a picture graph or bar graph, and interpret the results. This is an extension from first grade when students organized, represented, and interpreted data with up to three categories. They are able to use the graph selected to note particular aspects of the data collected, including the total number of responses, which category had the most/least responses, and interesting differences/similarities between the four categories. They then solve simple one-step problems using the information from the graph.

**Example:** The Second Graders were responsible for purchasing ice cream for an Open House event at school. They decided to collect data to determine which flavors to buy for the event. As a group, the students decided on the question, “What is your favorite flavor of ice cream?” and 4 likely responses, “chocolate”, “vanilla”, “strawberry”, and “cherry”.

The students then divided into teams and collected data from different classes in the school. Each team decided how to keep track of the data. Most teams used tally marks to keep up with the responses. A few teams used a table and check marks.

When back in the classroom, each team organized their data by totaling each category in a chart or table. Team A’s data was as follows:

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chocolate</td>
<td>12</td>
</tr>
<tr>
<td>Vanilla</td>
<td>5</td>
</tr>
<tr>
<td>Strawberry</td>
<td>6</td>
</tr>
<tr>
<td>Cherry</td>
<td>9</td>
</tr>
</tbody>
</table>

Measurement Data

Second Graders use measurement data as they move through the statistical process of posing a question, collecting data, analyzing data, creating representations, and interpreting the results. In second grade students represent the length of several objects by making a line plot. Students should round their lengths to the nearest whole unit.

Example: Measure 8 objects in the basket to the nearest inch. Then, display your data on a line plot.

Teacher: What do you notice about your data?
Student: Most of the objects I measured were 9 inches. Only 2 objects were smaller than 4 inches. I was surprised that none of my objects measured more than 9 inches!
Teacher: Do you think that if you chose all new objects from the basket that your data would look the same? Different? Why do you think so?

-From North Carolina, Unpacking Standards, Grade 2

Unit 9 - Measuring Length and Time Omit 1.1
Unit 4 - Pockets, Teeth and Favorite Things Investigation 1: Omit 1.2, 1.3.1.7
Differentiation Guide: pages 50-51 and 102-113
<table>
<thead>
<tr>
<th>Concepts developed in this unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves the iteration of units. They recognize that the smaller the unit, the more iteration they need to cover a given length. Students will also represent and interpret data by drawing picture and bar graphs. Lastly, students will be able to tell and write time to the nearest five minutes and will know the relationships of time including seconds in a minute, minutes in an hour, hours in a day, days in a week, a month and a year.</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Prior knowledge expected:</th>
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<tbody>
<tr>
<td><strong>Measurement and Data</strong></td>
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</table>

- Measure and estimate lengths in standard units.
- Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (2.MD.1)
- Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (2.MD.2)
- Estimate lengths using units of inches, feet, centimeters, and meters. (2.MD.3)
- Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (2.MD.4)

- Relate addition and subtraction to length.
- Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (2.MD.4)
- Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, . . . , and represent whole-number sums and differences within 100 on a number line diagram. (2.MD.6)

- Represent and interpret data.
- Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (2.MD.9)
- Draw a picture graph and bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems (see common addition and subtraction situations table in the MA Curriculum Frameworks p. 183) using information presented in a bar graph. (2.MD.10)

<table>
<thead>
<tr>
<th>Learning Outcomes:</th>
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<tbody>
<tr>
<td>Measure lengths indirectly and by iterating length units.</td>
</tr>
<tr>
<td>1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.</td>
</tr>
<tr>
<td>1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.</td>
</tr>
<tr>
<td>1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks.</td>
</tr>
<tr>
<td>1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.</td>
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</tbody>
</table>

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Boston Public Schools Elementary Mathematics Department
Grade 2 Scope and Sequence, 2015-2016. Last updated 8.12.15

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<table>
<thead>
<tr>
<th>MA 2011</th>
<th><strong>After completing each investigation, students will be able to:</strong></th>
<th>Days</th>
<th>Primary Curriculum Resource</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.MD.1</td>
<td>• Use a variety of tools to measure accurately.</td>
<td>8</td>
<td>Gr. 2., Unit 9: <em>Measuring Length and Time</em></td>
</tr>
<tr>
<td>2.MD.2</td>
<td>• Compare two lengths.</td>
<td></td>
<td>Omit 1.1</td>
</tr>
<tr>
<td>2.MD.3</td>
<td>• Use direct and indirect comparison to identify equal length.</td>
<td></td>
<td>1.2 Scavenger Hunt Workshop</td>
</tr>
<tr>
<td>2.MD.4</td>
<td>• Estimate using body benchmarks.</td>
<td></td>
<td>1.3 Measuring with Different Units</td>
</tr>
<tr>
<td>2.MD.6</td>
<td>• Read and interpret measurement data on a line plot.</td>
<td></td>
<td>1.4 Measuring Jumps</td>
</tr>
<tr>
<td></td>
<td><strong>Use academic language, such as:</strong></td>
<td></td>
<td>1.5 Comparing Jumps</td>
</tr>
<tr>
<td></td>
<td>length, width, measure, estimate, height, unit</td>
<td></td>
<td>1.5 will be extended over 4 days:</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Day 1 – Do 1.5. However give students 2 towers of 10 to measure and practice iterating. Also include SAB p. 17.</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Day 2 – Instruct students how to make a line plot and then have students make a line plot for 1 of types of jumps (frog, kid, or rabbit) of the class data (refer to Instructional note).</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Days 3 &amp; 4 - Create a line plot of another class’ data. Then have students describe the data of the two classes. May also want to have students record their findings. Include assessment from 1.6)</td>
</tr>
<tr>
<td>2.MD.1</td>
<td>• Identify length and width as different dimensions of an object.</td>
<td>3</td>
<td>Gr. 2, Unit 9: <em>Measuring Length and Time</em></td>
</tr>
<tr>
<td>2.MD.2</td>
<td>• Use a ruler accurately and apply strategies for accurate measurement.</td>
<td></td>
<td>2.1 The Land of Inch</td>
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<tr>
<td>2.MD.4</td>
<td>• Begin to understand partial units and describe using language such as a little less than, a little more than.</td>
<td></td>
<td>2.2 Measuring with the Inch-Brick Tool</td>
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<tr>
<td></td>
<td><strong>Use academic language, such as:</strong></td>
<td></td>
<td>2.3 A Map of the Land of Inch</td>
</tr>
<tr>
<td></td>
<td>inch, measurement, accurate</td>
<td></td>
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</tr>
<tr>
<td>2.MD.1</td>
<td>• Use a variety of tools to measure accurately.</td>
<td>8</td>
<td>Gr.2, Unit 9 - <em>Measuring Length and Time</em></td>
</tr>
<tr>
<td>2.MD.2</td>
<td>• Use the term inches, feet, yards, and centimeters to describe the length of objects.</td>
<td></td>
<td>3.1 Assessment: The King’s Foot</td>
</tr>
<tr>
<td>2.MD.3</td>
<td>• Use a ruler as a standard measuring tool.</td>
<td></td>
<td>3.2 Rulers and Body Benchmarks</td>
</tr>
<tr>
<td>2.MD.4</td>
<td>• Read and interpret measurement data on a line plot.</td>
<td></td>
<td>Make sure that by the end of the wrap up students understand that an inch brick tool and a ruler both have iterations. Additionally a ruler marks units with numbers</td>
</tr>
<tr>
<td>2.MD.5</td>
<td><strong>Use academic language, such as:</strong></td>
<td></td>
<td>3.3 Measurement Strategies</td>
</tr>
<tr>
<td>2.MD.6</td>
<td>unit, ruler, tape measure, yardstick, yard, inch, foot,</td>
<td></td>
<td>Use standard units of measurement ONLY (inches, feet, yards)</td>
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<td></td>
<td>benchmark, metric system, centimeter, meter</td>
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<td>3.4 Moving to Metric</td>
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<td>3.5 Metric Measurement</td>
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<tr>
<td>Day 1: Extension on measurement and line plot</td>
<td>2.MD.9 2.MD.10</td>
<td>5</td>
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<tr>
<td>Day 2: Story problems about length (1 day)</td>
<td>• Create a line plot given measurement data.</td>
<td>Gr. 2, Unit 4 Pockets, Teeth and Favorite Things</td>
<td></td>
</tr>
<tr>
<td>See Appendix</td>
<td>• Draw a picture and bar graph to represent a data set with up to four categories.</td>
<td>1.1 Guess My Rule with People</td>
<td></td>
</tr>
<tr>
<td>Investigations and the Common Core State Standards Book 3.6A –</td>
<td>• Students will be able to solve addition and subtraction problems about a given bar or picture graph.</td>
<td>Use picture or bar graph only. Also include Activity 3 from 1.3 at end of lesson so you have information for 1.4.</td>
<td></td>
</tr>
<tr>
<td>This will be a Math Workshop. Center One - p. C124, Center 2 – C125 – C126 (follow directions on CC121), Center 3 – Fluency based your class needs, Center 4 – Collect and Spend $2.00. Do not use the student activity pages 49A and 49B. Elapsed time is not a second grade standard.</td>
<td>1.4 &quot;Favorite Things&quot;</td>
<td></td>
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<tr>
<td></td>
<td>Use academic language, such as: data, rule, representation, attribute, category, survey, questionnaire</td>
<td>1.4A Bar Graphs</td>
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<td></td>
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<td>1.4, 1.4A, 1.5 (using only picture or bar graph)</td>
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<td>1.5 Organizing &quot;Favorite Things&quot; Data</td>
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<td>1.6 Sharing &quot;Favorite Things&quot; Data</td>
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<td>Omit 1.7</td>
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<tr>
<td>No End-of-Unit Assessment</td>
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